Therefore the projectile will fall without the circle, if $\sin\varphi$ is less than $\frac{1}{2}(1+\cos\theta)^{\frac{1}{2}}-\frac{1}{2}(1-\cos\theta)^{\frac{1}{2}}$; but will fall within if $\sin\varphi$ is greater than $\frac{1}{2}(1+\cos\theta)^{\frac{1}{2}}+\frac{1}{2}(1-\cos\theta)^{\frac{1}{2}}$.

If all possible directions are equally probable, the chance of the projectile falling within the circle is $1-(1-\cos\theta)^{\frac{1}{2}}=1-\sqrt{2}\sin\frac{1}{2}\theta$.

Hence the required chance is

$$p=\frac{\int_0^{\frac{1}{2}\pi}(1-\sqrt{2}\sin\frac{1}{2}\theta)d\theta}{\int_0^{\pi}d\theta}=\tfrac{1}{2}-(2/\pi)(\sqrt{2}-1).$$

III. Solution by GEORGE LILLEY, Ph. D., Professor of Mathematics, University of Oregon, Eugene, Ore.

The diameter of the field is v^2/g, and the range must not exceed $(v^2/g)\cos\phi$. The elevation may vary from $0°$ to θ_1, and from $\frac{1}{2}\pi-\theta_1$ to $\frac{1}{2}\pi$ for each value of ϕ, where θ_1 is determined by $\sin2\theta_1=\cos\phi$, ϕ the azimuth of the projectile measured from the diameter, θ the elevation of the gun, v the velocity of projection, and g the intensity of gravity.

The surface-element of the enveloping hemisphere whose radius is R or v^2/g is $R^2\cos\theta d\theta d\phi$.

Using only one-half the field and one-fourth of the sphere the required chance is

$$\frac{R^2\int_0^{\frac{1}{2}\pi}\int_0^{\theta_1}\cos\theta d\theta d\phi+R^2\int_0^{\frac{1}{2}\pi}\int_{\frac{1}{2}\pi-\theta_1}^{\frac{1}{2}\pi}\cos\theta d\theta d\phi}{\pi R^2}$$

$$\text{or } \frac{1}{\pi}\int_0^{\frac{1}{2}\pi}(1-\sqrt{2}\sin\tfrac{1}{2}\phi)d\phi, \text{ or } \tfrac{1}{2}-\frac{2}{\pi}(\sqrt{2}-1).$$

Solved in a similar manner by *L. C. WALKER*. Solved with different results by *HENRY HEATON*, and *P. H. PHILBRICK*.

75. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy, Irving College, Mechanicsburg, Pa.

Find the mean area of all plane rectilineal right triangles having a constant perimeter p.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let x and y be the base and altitude, respectively.

Then $x+y+\sqrt{(x^2+y^2)}=p$, the perimeter .....(1).

$$\therefore x=\frac{p^2-2py}{2(p-y)}. \quad \therefore \text{Area}=\frac{p}{4}\frac{(p-2y)y}{p-y}.$$

The limits of y are $y=0$ to $y=x$. When $y=x$ in (1) we get $y=p/(2+\sqrt{2})=y'$.

$$\therefore \triangle=\frac{p}{4}\frac{\int_0^{y'}\frac{(p-2y)y}{p-y}dy}{\int_0^{y'}dy}=\frac{2+\sqrt{2}}{4}\int_0^{y}(2y+p-\frac{p^2}{p-y})dy$$

$$=\frac{2+\sqrt{2}}{4}\left[\frac{1}{(2+\sqrt{2})^2}+\frac{1}{2+\sqrt{2}}+\log\left(\frac{1+\sqrt{2}}{2+\sqrt{2}}\right)\right]p^2$$

$$=\frac{p^2}{4}\left[\frac{1}{2+\sqrt{2}}+1+(2+\sqrt{2})\log\left(\frac{1}{\sqrt{2}}\right)\right]$$

$$=\frac{p^2}{8}\left[4-\sqrt{2}-(2+\sqrt{2})\log 2\right].$$

II. Solution by J. SCHEFFER, A. M., Hagerstown, Md., and L. C. WALKER, Associate Professor of Mathematics, Leland Stanford University, Palo Alto, Cal.

Denote the base of the right triangle by x, and the perpendicular by y. Then the hypotenuse is $p-x-y$, and from $(p-x-y)^2=a^2+y^2$, we obtain

$y=\dfrac{p^2-2px}{2(p-x)}$. $\therefore$ The area$=\dfrac{p}{4}\dfrac{(p-2x)x}{p-x}$, and the average area is

$$\frac{p}{4}\int_0^{\frac{1}{2}p}\frac{(p-2x)x}{p-x}dx\div\int_0^{\frac{1}{2}p}dx=\tfrac{1}{2}\int_0^{\frac{1}{2}p}\frac{(p-2x)x}{p-x}dx=\tfrac{1}{2}\int_0^{\frac{1}{2}p}\left(2x+p-\frac{p^2}{p-x}\right)dx$$

$$=\tfrac{1}{8}p^2(3-4\log 2).$$

NOTE.—The difference in the results of these two solutions is due to the different limits assumed for the variable. It seems that the proper limits to assume for the variable x is 0 and $\frac{1}{2}p$, for in this way, and in this way only, do we get the totality of triangles according to our assumed law of distribution of the triangles. The law of distribution tacitly assumed in both solutions is that the number of triangles is proportional to the base of the triangle. Had some other law of distribution been assumed, different results from either of the above would have been obtained. The problem is stated in the indefinite form because the law of distribution is not stated. See Dr. E. H. Moore's Note on Mean Values, Vol. II, page 303, of THE AMERICAN MATHEMATICAL MONTHLY. ED. F.